Combinatorics and Graph Theory III
Tutorial 3

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1 K_k-minor free graphs

Lemma 1.1. *For $k \leq 4$, a graph is K_k-minor free if and only if it can be obtained from copies of $K_1, K_2, \ldots, K_{k-1}$ by clique-sums.*

Lemma 1.2. *A graph is K_5-minor free if and only if it can be obtained from planar graphs and copies of the Wagner graph by clique-sums.*

1. Prove that for $k \leq 4$, K_k-minor-free graphs are $(k-1)$-degenerate.
2. Prove that K_5-minor-free graphs are 5-degenerate.
3. What is the maximum number of edges of an n-vertex K_k-minor-free graph for $k \leq 5$?
4. Prove that $K_{3,3}$-minor-free graphs are exactly the graphs obtained from planar graphs and copies of K_5 by clique-sums.
5. What are examples of graphs not containing K_k as a topological minor yet with minimum degree $\Omega(k^2)$?